Guaranteed Matrix Completion under Multiple Linear Transformations





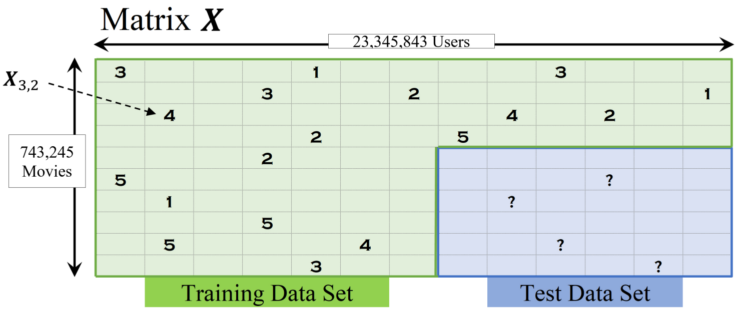
Chao Li[†] Wei He[†] Longhao Yuan[‡†] Zhun Sun[†] Qibin Zhao[†§]

Email Address: `chao.li@riken.jp`

Poster ID:162

[†]RIKEN-AIP, Japan
[‡]Saitama Institute of Technology, Japan
[§] Guangdong University of Technology, China

What is Matrix Completion (MC)?



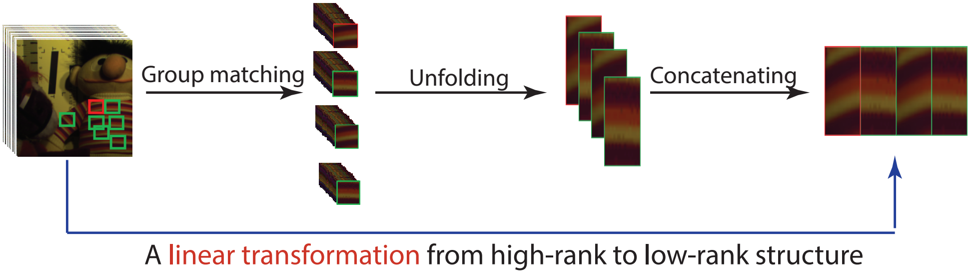
Advantage

There is theoretical guarantee to bound the reconstruction error.

Key Assumption

The reconstructed matrix is *low-rank*.

An example – Non-local Trick in Image Restoration



Summary

A significant low-rank structure appears under some transformations.

Problem

The conventional theoretical analysis for guarantee is no longer suitable.

Our Work – Formulation

Our work is to formulate and complete the framework of this problem.

We generalize the problem as *Matrix Completion under Multiple linear-Transformations (MCMT)*:

$$\min_{\mathbf{X}\in\mathbb{R}^{M_1\times M_2}} \frac{1}{2}\|P_\Omega(\mathbf{X}) - P_\Omega(\mathbf{Y})\|_F^2 + \lambda \sum_{i\in[K]} \|\mathcal{Q}_i(\mathbf{X})\|_*. \tag{1}$$

$\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{M_1\times M_2}$ - the target matrix and its observation;
$P_\Omega : \mathbb{R}^{M_1\times M_2} \to \mathbb{R}^{M_1\times M_2}$ - sampling projection;
$\mathcal{Q}_i : \mathbb{R}^{M_1\times M_2} \to \mathbb{R}^{N_1^{(i)}\times N_2^{(i)}}$ - linear transformations for each $i \in [K]$.

Note

$\mathcal{Q}_i$ can be represented by a 4th-order tensor, *i.e.* $\mathcal{Q}_i \in \mathbb{R}^{M_1\times M_2\times N_1^{(i)}\times N_2^{(i)}}$

Our work – Main Contribution

Theorem

With some assumptions on the $\mathcal{Q}_i, i \in [K]$, *and further assume that the tuning parameter satisfies* $\lambda > \|P_\Omega(\eta)\|_2/\sqrt{M}$. *Then the reconstruction error is upper-bounded by*

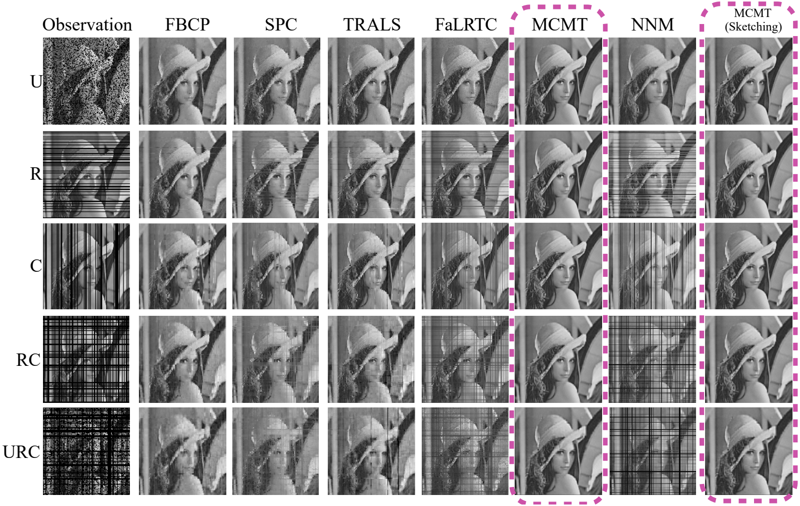
$$\|\hat{\mathbf{M}} - \mathbf{M}_0\|_F \le \mathcal{O}\left(\lambda \cdot M^{0.5} \frac{\delta_{\max}(\{\mathcal{Q}_i\})}{\delta_{\min}(\{\mathcal{Q}_i\})} \left(K^2 + M^{K-0.5} \delta_{\max}(\{\mathcal{Q}_i\})\right)\right), \tag{2}$$

where $\delta_{\max}(\cdot)$ *and* $\delta_{\min}(\cdot)$ *denotes the maximum and the non-zero minimum singular values from all* $\mathcal{Q}_i$*'s, respectively.*

Remark

The upper-bound of the reconstruction error is linearly controlled by the condition number of the transformations.

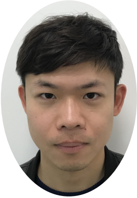
Illustrative Experiment



Guaranteed Matrix Completion under Multiple Linear Transformations


Chao Li[†]


Wei He[†]


Longhao Yuan[‡†]


Zhun Sun[†]


Qibin Zhao[†§]

Email Address: `chao.li@riken.jp`

Poster ID:162

[†]RIKEN-AIP, Japan
[‡]Saitama Institute of Technology, Japan
[§] Guangdong University of Technology, China