Reshuffled Tensor Decomposition and Exact Recovery of Low-rank Components

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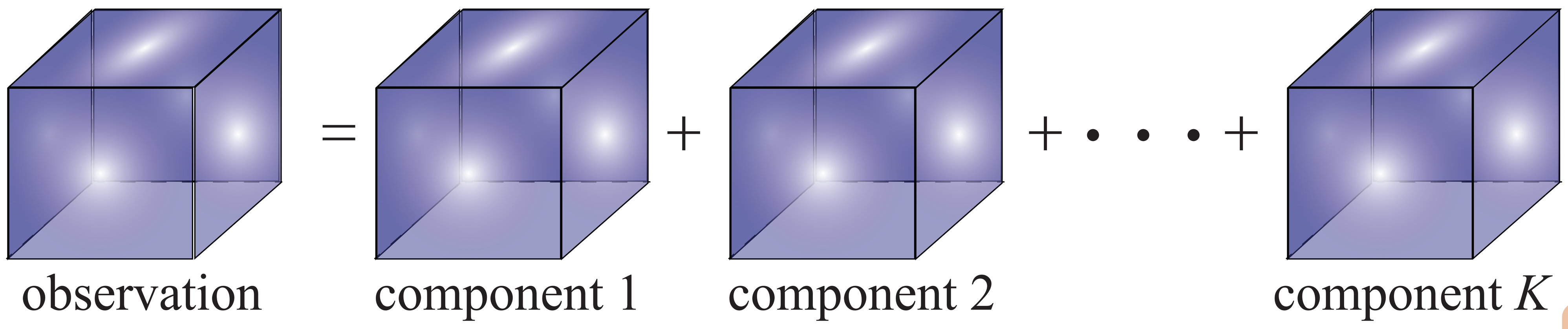
Motivation

We consider a linear inverse problem from the generative model:



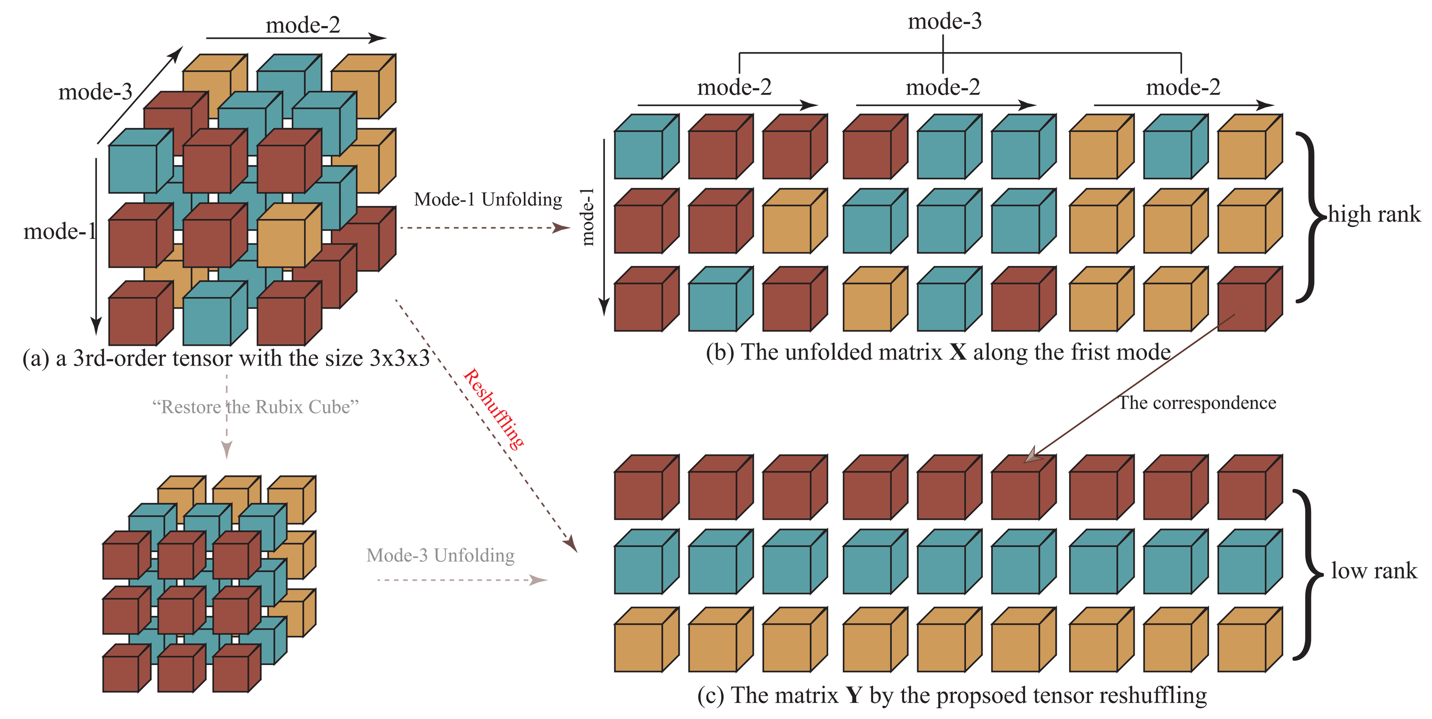
We further assume:
Each component has the low-rank structure under reshuffling.

1. What is reshuffling?
2. Why we need reshuffling?

The key motivation:
Is it possible to exactly recover the components from the observation under this assumption?

Reshuffling Operation

Reshuffling is an extension of tensor unfolding.



Theoretical Results

Incoherence measurement for each component:

$$\mu_i(\mathcal{A}) := \max_{j\neq i} \max_{\substack{\mathcal{Y}\in\mathbb{T}_{\mathbb{P}_i(k_i)}(\mathcal{A}),\\ \|R_i(\mathcal{Y})\|_2\leq 1}} \|R_j(\mathcal{Y})\|_2,$$

Theorem (Exact-Recovery condition): The minimizer of Reshuffled-TD equals the true components from the generative model when

$$\max_{i=1,\ldots,N} \mu_i(\mathcal{A}_i) < \frac{1}{3K-2}.$$

Algorithm

Reshuffled-TD:

$$\min_{\mathcal{A}_1,\ldots,\mathcal{A}_N} \sum_{i=1}^{N} \|R_i(\mathcal{A}_i)\|_*, \quad \text{s.t.,}\ \mathcal{X} = \sum_{i=1}^{N}\mathcal{A}_i,$$

↳ reshuffling operation

Algorithm 1 Reshuffled-TD via ADMM

Initialize: The observation $\mathcal{X}$, the Lagrangian dual $\mathcal{Y} = sgn(\mathcal{X})$, $\mathcal{A}_i = \mathcal{X}/K, \forall i$, and $\rho > 1, \kappa_0 > 0$

Iteration until convergence:

//STEP 1: update every component $\mathcal{A}_i$, $i = 1,\ldots,N$

for $i = 1,\ldots,K$ **do**

$\mathcal{A}_i \leftarrow R_i^{-1}\left(D_{\kappa^{-1}}\left(R_i\left(\mathcal{X} - \sum_{j\neq i}^{N}\mathcal{A}_j + \kappa^{-1}\mathcal{Y}\right)\right)\right)$

end for

//STEP 2: update the Lagrangian dual $\mathcal{Y}$

$\mathcal{Y} \leftarrow \mathcal{Y} + \kappa\left(\mathcal{X} - \sum_{i=1}^{K}\mathcal{A}_i\right)$

//STEP 3: update the scalar κ

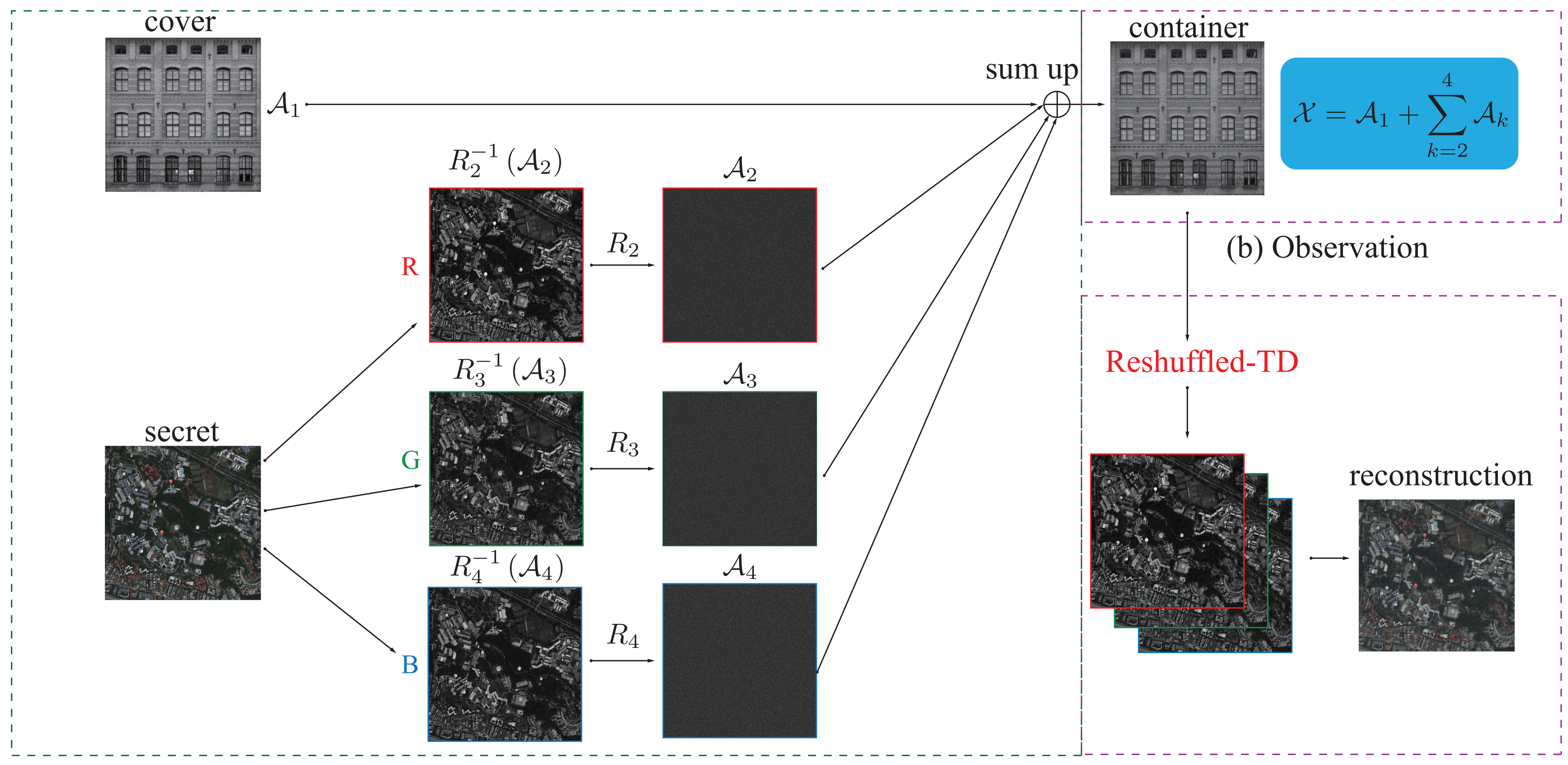
$\kappa \leftarrow \rho\kappa$

Output: $(\mathcal{A}_i,\ldots,\mathcal{A}_N)$

$D_\kappa(\cdot)$ denotes matrix soft-thresholding.

Image Steganography

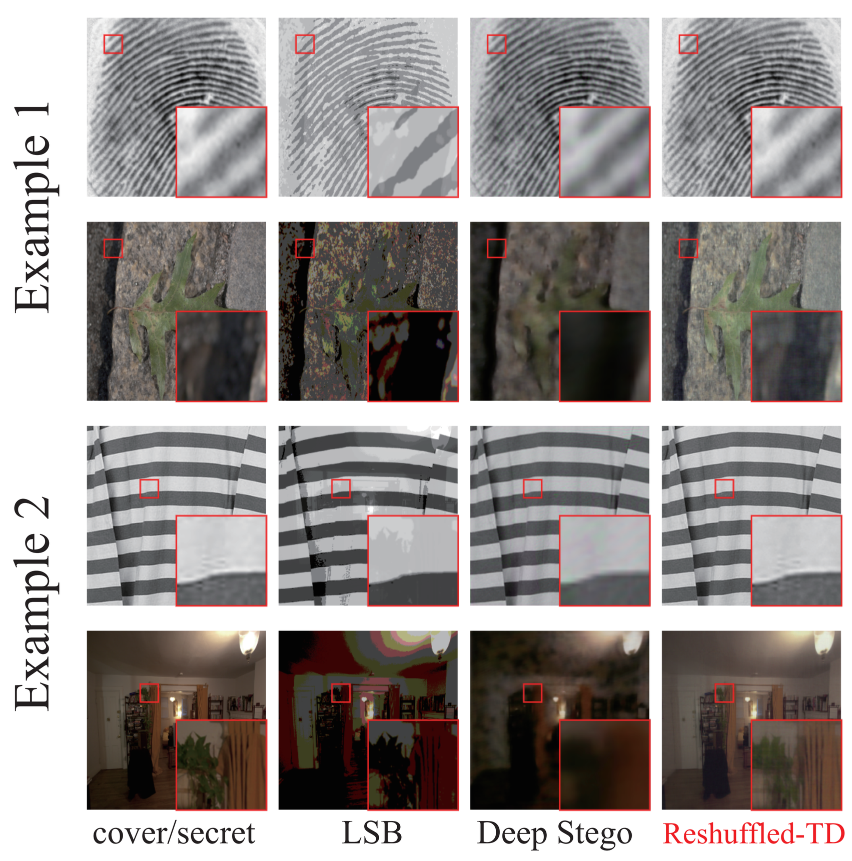
1. We consider both cover and secret image as the components.
2. Our theory guarantees that the secret image can be reconstructed.



(a) Generation (from components to observation) (c) Decomposition (inverse problem)

For brevity, we use random reshuffling in the experiment.

Experimental Results



Datasets	LSB 1 bit/chn	LSB 2 bits/chn	Reshuffled-TD $\sigma = 0.01$	Reshuffled-TD $\sigma = 0.05$	Reshuffled-TD $\sigma = 0.1$
DTD(C)	26.70	9.66	32.25	18.45	12.63
CART.(S)	6.92	14.42	13.13	22.67	25.92
DTD(C)	23.77	7.53	36.86	23.69	17.78
DTD(S)	3.38	7.84	4.71	11.36	13.12
DTD(C)	24.05	7.76	34.30	21.47	15.64
FIVEK(S)	1.12	6.00	4.91	11.62	13.54
FIVEK(C)	23.02	6.56	36.33	23.42	17.49
FIVEK(S)	3.37	7.52	-0.95	5.70	9.29
FVC(C)	18.19	3.27	33.54	20.25	14.40
FIVEK(S)	3.32	6.42	5.04	12.80	14.66
LIVE(C)	24.50	7.66	37.71	24.71	18.78
FIVEK(S)	4.08	7.58	4.75	11.49	12.91
Average	23.37	7.07	35.165	**22.00**	16.12
	3.70	8.30	5.26	**12.61**	14.91

About Us

Institute: ***Center for Artificial Intelligence Project***

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Interests:
machine learning, signal processing, computer vision

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